# STATEMENT REPORT

**under the procedure for acquisition of the scientific degree "Doctor of Science"**
by candidate **Assoc.Prof. Dr. Borislav Radkov Draganov,**
**of the DSci Thesis entitled: "Simultaneous approximation by the Bernstein operator .",**
in the Scientific field: **4. Natural Sciences, Mathematics and Informatics**
Professional field: **4.5. Mathematics**
**Faculty of Mathematics and Informatics (FMI), Sofia University "St. Kl. Ohridski" (SU),**


The statement report has been prepared by: Assoc.Prof. Dr. Irina Krasimirova Georgieva - Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,
as a member of the scientific jury for the defense of this DSci thesis according to Order № РД 38-627/ 28.11.2023 of the Rector of the Sofia University.


1. **General characteristics of the dissertation thesis and the presented materials**

   The presented dissertation thesis for Doctor of Sciences Degree in Mathematics "Simultaneous approximation with Bernstein operators" contains 178 pages and consists of an introduction and 6 chapters; 100 titles in the presented bibliography. There are presented 9 articles on which the dissertation is based, an abstract of the dissertation in Bulgarian and in English.


2. **Short CV and personal impressions of the candidate**

   Borislav Draganov defended his diploma thesis with supervisor prof. Kamen Ivanov in 1998, and in 2004, he defended his PhD thesis on "New method for characterization of K-functionals and application in approximation theory", again with supervisor prof. K. Ivanov. Since 2002 B. Draganov has been working at Faculty of mathematics and Informatics - Sofia University "St. Kliment Ohridski", first as an assistant professor and since 2011 until now as an associate professor. Ass. Prof. Dr. Draganov has vast teaching experience. Since 2004 B. Draganov has been also working in the Department "Mathematical Modeling and Numerical Analysis", Institute of Mathematics and Informatics, Bulgarian Academy of Sciences.

   The interests of Assoc. Dr. Borislav Draganov are mainly in the field of approximation theory. He works on establishing accurate and feasible error estimates of approximation processes generated by linear or nonlinear operators, and best approximations with algebraic or trigonometric polynomials in different functional spaces. Also he works with generalized

Fourier series, sampling series, orthogonal polynomials, multipliers, and applications of Fourier analysis to approximation theory.

I have known Borislav Draganov since his student years. He is erudite, hard-working, also helpful as a colleague. It has always been a pleasure to discuss with him various mathematical problems of common interest.

3. **Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure**

The present dissertation thesis "Simultaneous Approximation with Bernstein Operators" is based on results presented in 9 papers [16-24] (numbering relative to English version of the abstract). It is a classical result that Bernstein polynomials approximate not only the function $f$, but also its derivatives $f^{(s)}$. For $f \in C^s[0,1]$ it holds that $\lim_{n \to \infty} B_n f^{(s)}(x) = f^{(s)}(x)$ uniformly in [0,1]. That phenomenon is reffed as simultaneous approximation. This dissertation is devoted to finding estimates of the rate of simultaneous weighted approximations of a continuous function and its derivatives by the Bernstein operator, estimates of the rate of simultaneous weighted approximations with iterated Boolean sums of Bernstein operators, and estimates of the rate of simultaneous weighted approximations by two kinds of Bernstein polynomials with integer coefficients. Direct and weak converse inequalities of Voronovskaya type are derived. The dissertation consists of an introduction and 6 chapters.

Chapter 1 gives definitions and basic properties of classical moduli of smoothness and K-functionals, as well as their generalizations with weights (introduced by Ditzian-Totik, 1987), which have the advantage of accounting more accurately for the approximation at the ends of the interval, a property that approximations with Bernstein operators have.

In Chapter 2, a number of inequalities are proved-between weighted supremum-norms of different derivatives of a function, and also inequalities between weighted supremum-norms of derivatives of a functions and weighted supremum-norms of the values of a special differential operator which arises naturally in approximations with the Bernstein operator, and also in the more general case of approximations with iterated Boolean sums of the Bernstein operator. The results of this chapter (published in papers [17, 18, 24]) are interesting in their own right, but are also used extensively later in the dissertation.

In Chapter 3, the direct (Theorem 3.3, Theorem 3.5) and (corresponding) strong converse estimates (Theorem 3.8, Theorem 3.26) of the rate of simultaneous approximation by the Bernstein operator in a supremum-norm with Jacobi weights ( then it follows that the direct inequalities are exact) are proved. The weighted Ditzian-Totik (1987) moduli of smoothness are used. The K-functionals $K_s^D(f,t)_{w.}$ are used to characterize the rate of approximation .

It follows from Theorems 3.3 and 3.26 that the weighted supremum-norm of the error of the simultaneous approximation with the Bernstein operator is equivalent to the K-functional, i.e. $\left\|w(B_n f - f)^{(s)}\right\| \sim K_s^D(f^{(s)}, n^{-1})_w$ (Theorem 3.30). We also obtain a characterization of rate of the simultaneous approximation by the Kantorovich operator-Theorem 3.41 and a characterization of the rate of approximation by the Kantorovich operator-Theorem 3.44. The results of this chapter are published in [18, 19].

In Chapter 4, generalizations of the results of Chapter 3 are proved for the case of iterated Boolean sums of the Bernstein operator $B_{r,n} := I - (I - B_n)^r$, and also for iterated Boolean sums of the Kantorovich operator. The results obtained in this chapter are very interesting since the iterated Boolean sums of the Bernstein operator $B_{r,n}$ have saturation order $n^{-r}$, while the operator $B_n$ has saturation order $n^{-1}$. The results of this chapter are published in papers [16, 17, 18, 22, 23].

In Chapter 5, direct (Theorem 5.1 and Theorem 5.4) and weak converse (Theorem 5.5) error estimates for the simultaneous approximation by two modifications of Bernstein polynomials, which are algebraic polynomials with integer coefficients are proved. The corresponding operators are not linear. The results presented in this chapter are published in papers [20, 21].

In Chapter 6, direct and weak converse inequalities of Voronovskaya type for the Bernstein operator are obtained (Theorem 6.1). The rate of convergence in Voronovskaya's Theorem is characterized in Theorem 6.4 and Corollary 6.5. The results of this chapter are published in [24].

## 4. Approbation of the results

The results obtained in this dissertation thesis are original and have been published in 9 papers (6 of them with impact factor) in reputable scientific journals - Journal of Approximation Theory, Results in Mathematics, etc. (refereed in Web of Science and Scopus) and conference proceedings (ref. in Zentralblatt and Mathematical Reviews) and have undergone a peer-review process. So far, these papers have 15 independent citations and most of the citations are in publications in Scopus and Web of Science, there are also two citations in theses (abroad), which shows that the work of Assoc. Prof. Dr. Borislav Draganov is known not only to mathematicians in Bulgaria, but is also used in the works of authors around the world.

Paper [24] is joint work with Assoc. Prof. Dr.Ivan Gadzhev, and the contributions of both authors are equal. The scientific results of the dissertation thesis are presented

at international conferences. Moreover, Assoc. Prof. Dr. Draganov presented his results in two plenary talks by invitation at international conferences in 2021 and 2022.

It is important to note that:

- the scientific works meet the minimum national requirements (under Art. 2b, para. 2 and 3 of ADASRB*) and respectively to the additional requirements of Sofia University "St. Kliment Ohridski" for acquiring the scientific degree "Doctor of Science"in the scientific field and professional field of the procedure;

-  the results presented by the candidate in the dissertation work and scientific works to it do not repeat such from previous procedures for acquiring a scientific title and academic position;

- there is no plagiarism proven in the legally established order in the submitted dissertation work and scientific papers under this procedure.

5.  **Qualities of the abstract**

The abstract consists of 36 pages in Bulgarian and 35 pages in English and meets all the requirements, correctly presenting the results and content of the dissertation.

6. **Critical notes and recommendations**

I have no critical remarks. I only noticed a few typos.

7. **Conclusion**

This dissertation contains original, very significant results that solve classical problems in approximation theory. The great teaching talent of Assoc. Prof. Borislav Draganov - the results obtained by him are presented in the most comprehensive way despite their complexity, depth and technical difficulty.

Having become acquainted with the DSci thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented DSci thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of the scientific degree "Doctor of Science" in the Scientific field **4. Natural Sciences, Mathematics and Informatic**s, Professional field **4.5 Mathematics (Mathematical Analysis).** In particular, the candidate meets the minimal national requirements in the

professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, **I strongly recommend** the scientific jury to award **Assoc. Prof. Dr. Borislav Radkov Draganov**, the scientific degree „Doctor of Science" in the   Scientific field **4. Natural Sciences, Mathematics and Informatic**s, Professional field **4.5 Mathematics (Mathematical Analysis)**.

Date: 22.02.2024                              Signature:

Assoc.Prof. Dr. Irina Krasimirova Georgieva